

CRP RELEASE 1

Introduction

Welcome to the inaugural **Collaborative Research Project (CRP)**. Over the next month, twenty-four teams from twelve states and the District of Columbia will be collaborating to investigate *Tropical Mathematics*.

Dr. David Speyer, Associate Professor of Mathematics at the University of Michigan, and Dr. Bernd Sturmfels, Professor of Mathematics, Statistics and Computer Science at the University of California Berkeley provide an invitation to this research area in the article *Tropical Mathematics* which later appeared in the June 2009 Mathematics Magazine.

In the CRP you will investigate a problem which arises in this popular account. The scope of your contribution may extend beyond the primary challenge described below, but it should offer a starting point for your investigation. Periodically we will be offering information releases to guide your exploration. The releases will be announced through the CRP twitter feed @CRPmathematics. If your group is moving at a faster pace you are encouraged to move beyond the specific questions posed in any particular release.

Background (Days 1-3)

Begin by reading the article *Tropical Mathematics* from the June 2009 edition of Mathematics Magazine (available through your college's library resources or the author's webpage). When reading, challenge your team to construct new examples and pose additional questions. Use the Wiki's *Survey Article Questions* page to share your questions.

If you find or create resources which might be of interest to the project, please link to these *Additional Resources* on the Wiki. You should take time to get comfortable with the material in this paper before moving forward in the research process.

Collaboration

You are encouraged to connect with teams from institutions which are different from your own. Try to connect with at least two institutions which were not on your radar when applying to college.

A project Wiki is available at <http://crpwiki.math.wfu.edu/mediawiki>. Consider asking each other questions about the reading, or presenting sections of the reading to a larger group over Skype. To help identify other teams to work with, you might post information about your teams background and experiences on Twitter or on your *talk* page on the Wiki. Also consider using other collaborative platforms which meet your needs (google hangouts, dropbox). The Wiki contains a *Calendar* page where you can post events that might be of interest to other participants.

Primary Challenge

The CRP will focus on applications of Tropical Mathematics to the area of phylogenetics. In particular we seek to address the Research Problem stated at the top of page 171 of the Tropical Mathematics paper.

Definition.

We say that a metric D has phylogenetic rank $\leq k$ if there exist k tree metrics $D_{i,j}^{(1)}, D_{i,j}^{(2)}, \dots, D_{i,j}^{(k)}$ such that

$$D_{i,j} = \text{Max}(D_{i,j}^{(1)}, D_{i,j}^{(2)}, \dots, D_{i,j}^{(k)}) \text{ for all } 1 \leq i, j \leq n.$$

Equivalently, the matrix $X = -D$ is the tropical sum of the matrices $X^{(i)} = -D^{(i)}$:

$$X = X^{(1)} \oplus X^{(1)} \oplus \dots \oplus X^{(k)}.$$

The aim of the notion of phylogenetic rank is to model distance data that is a mixture of k different evolutionary histories.

Research Problem. *The set of metrics of phylogenetic rank $\leq k$ is a polyhedral fan in $\mathbb{R}^{\binom{n}{2}}$. Compute this fan, and explore its combinatorial, geometric and topological properties, especially for $k = 2$.*